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PRESSURE IN A SHOCK WAVE WITH AN INTENSE SPARK DISCHARGE

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SHOCK WAVE PRESSURE WITH AN INTENSE SPARK DISCHARGE IN WATER

/ Following is the translation of an article by I.M. Astrakhan entitled "Davleniye na Udarnoy Volne pri Sil'nom Iskrovom Razryade v Vode"(English version above) in Izvestiya Vysshi'h Uchebnykh Zavedeniy - Neft' i Gaz (News of Higher Institutions of Learning - Petroleum and Gas), No. 10, Baku, 1959, pages 87 - 92 /

A method is explained in this article for computing the pressure at a shock wave with a spark discharge in water, based on the solution of a self-modeled problem of a strong detonation in a compressible medium.

An underwater spark discharge is widely used as a souce of pressure in water (1). The problem of the strong point explosion in an ideal gas and in an uncompressible fluid has been solved by L.I. Ledov (2). The self-modeled / singular / problem of a point explosion in a compressible fluid, like water, has been examined by N.N. Kochina and N.S. Mel'nikova (3). S.A. Khristianovich has examined the shock wave in water, where the pressure at the wave front did not exceed 300 atm. (4).

At short distances from the discharge channel axis the shock wave is propagated with cylindrical symmetry corresponding to the symmetry of the discharge, while at large distances with spherical symmetry, as from a point source. It is in connection with this that we have solved the self-modeled problem of the detonation with a cylindrical wave and converted S.A. Khristianovich's solution for an explosive to the case of a spark discharge. In the intermediate region we will use the method of graphic interpolation.

As it has been shown in (1), in order for the problem of the strong detonation in a compressible medium to be self-modeled, it will suffice that the internal energy of the medium

$$\varepsilon(p, \rho) = \frac{p}{\rho_0}\varphi\left(\frac{\rho}{\rho_0}\right) + \text{const}, \tag{1}$$

where φ - is an arbitrary function of its argument. In this case the adiabatic equation

$$p = \psi(S)\varkappa\left(\frac{\rho}{\rho_0}\right), \tag{2}$$

where $\psi(S)$ - is a certain function of entropy S. The connection between $\varphi(R)$ and $\varkappa(R)$ is determined by the formulas

$$\varphi(R) = \frac{1}{\varkappa(R)}\left(C + \int\frac{\varkappa(R)}{R^2}\,dR\right);$$

$$\varkappa(R) = \frac{C}{\varphi R}\exp\int\frac{dR}{R^2\varphi(R)}, \tag{3}$$

where C - is an arbitrary constant,

$$R = \frac{\rho}{\rho_0}.$$

The equations of one-dimensional, nonsteady motion of the ideal compressible medium have the form

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0,$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial r} + (\nu - 1)\rho\frac{v}{r} = 0, \tag{4}$$

$$\frac{\partial}{\partial t}\frac{p}{\varkappa\left(\frac{\rho}{\rho_0}\right)}+v\frac{\partial}{\partial r}\frac{p}{\varkappa\left(\frac{\rho}{\rho_0}\right)}=0,$$

where $\nu = 1$ for plane waves, $\nu = 2$ for cylindrical and $\nu = 3$ for spherical waves.

We will consider that at moment $t = 0$ an explosion occurs (i.e. a finite energy E_0 is instantaneously released) in a resting compressible medium at the center of symmetry and that the detonation is intense (i.e. the pressure p_0 in the undisturbed medium can be neglected in comparison with the pressure at the shock front). In crossing the surface of the intense explosion the conditions of conservation of mass, angular momentum and energy flow should be fulfilled. These conditions are the limits for equations (4):

$$\begin{gathered}
-\rho_1 c=\rho_2(v_2-c),\\
\rho_1 c^2=p_2+\rho_2(v_2-c)^2,\\
\frac{1}{2}c^2=\frac{1}{2}(v_2-c)^2+\frac{p_2}{\rho_2}+\frac{p_2}{\rho_2}\varphi\left(\frac{\rho_2}{\rho_0}\right),
\end{gathered} \tag{5}$$

where c - is the velocity of the shock wave.

Thus, there enter into the conditions of the problem only two constants with independent dimensions: ρ_0 and E_0

$$[\rho_0]=ML^{-3},\quad [E_0]=ML^{\nu-1}T^{-2}.$$

Consequently, the problem becomes self-modeled (1). We will assume

$$\begin{gathered}
v=\frac{r}{t}V(\lambda),\\
\rho=\rho_0 R(\lambda),\\
p=\rho_0\frac{r^2}{t^2}p(\lambda),\quad \left(\lambda=\frac{r}{r_2}\right),
\end{gathered} \tag{6}$$

where r_2 - is the radius of the shock wave.

For cylindrical symmetry

$$r_2 = \left(\frac{E}{\rho_0}\right)^{1/4} t^{1/2} \qquad (7)$$

The velocity of the shock front

$$c = \frac{1}{2}\left(\frac{E}{\rho_0}\right)^{1/4} t^{-1/2} = \frac{1}{2}\sqrt{\frac{E}{\rho_0}}\, r_2^{-1}, \qquad (8)$$

where E - is a certain constant with energy dimensions proportional to the energy of the explosion

$$E_0 = \alpha E .$$

From (5), (6) and (7) we obtain

$$R_2\varphi(R_2) = \frac{1}{2}\left(1 - \frac{R_1}{R_2}\right),$$

$$V_2 = \frac{1}{2}\left(1 - \frac{R_1}{R_2}\right),$$

$$P_2 = \frac{1}{2} R_1 V_D. \qquad (9)$$

The first of these conditions gives

$$R_1 = \frac{R_2}{1 + 2R_2\varphi(R_2)} . \qquad (10)$$

At the shock wave we obtain

$$\rho_2 = \rho_1\left[1 + 2R_2\varphi(R_2)\right],$$

$$v_2 = \frac{2cR_2\varphi(R_2)}{1 + 2R_2\varphi(R_2)},$$

$$p_2 = \frac{2\rho_1 c^2 R_2\varphi(R_2)}{1 + 2R_2\varphi(R_2)}. \qquad (11)$$

Usually the equation of the state of water is presented in the form (2)

$$p = \psi(S)(\rho^n - \rho_0^n) . \qquad (12)$$

Then $\varkappa(R) = R^n - 1$ and from (3) we obtain an expression for O R which can to a certain degree of accuracy approximate the function

$$\varphi(R) = \frac{R-1}{2R^2}. \qquad (13)$$

We now compute the pressure at the shock wave. We assume

$$\rho_0 = 0.938 \text{ g/cm}^3, \quad \rho_1 = 0.999 \text{ g/cm}^3. \qquad (14)$$

This means $R_1 = 1.066$, $R_2 = 1.816$. (15)

Under these conditions $\alpha = 0.01$.

Considering (10), (11) and (13) we obtain

$$p_2 = \rho_2 c^2 R_1 \frac{R_2 - 1}{R_2^2} = \frac{\rho_1 c^2 (R_2 - 1)}{2R_2 - 1}.$$

From (8) $$p_2 = \frac{\rho_1 E_0 (R_2 - 1)}{4\alpha \rho_0 (2R_2 - 1) r_2^2}. \qquad (16)$$

Substituting the data from (14) and (15) here, we obtain

$$p_2 = 8{,}2 \frac{E_0}{r_2^2}. \qquad (17)$$

We will compute the detonation energy E_0 for the data of an experiment made by the Chair of "Trade Machines and Mechanisms" of Moscow Institute of the Petroleum and Gas Chemical Industries imeni Academician I.M. Gubkin. When $C = 1.5\ \mu f$, $U = 50$ kv, $d = 6$ cm. (18)

where C - the capacity of the capacitor; U - is the voltage at the electrodes; d - is the distance between the electrodes, we will assume

$$E_0 = \frac{CU^2}{2d} = 0.31 \cdot 10^{10} \text{ erg/cm}. \qquad (19)$$

Thus, according to (17)

$$p_2 = 0{,}28 \cdot 10^5 r_2^{-2} \text{ atm}. \qquad (20)$$

The results of computing p_2 by formula (20) are as follows:

r_2, cm = 3; 5; 8;

p_2, atm = 3000; 1133.3; 442.6.

At considerable distances from the place of the explosion the shock wave is propagated with spherical symmetry. With low pressures the equation of state for water is taken in the form (3)

$$p = B\left[\left(\frac{\rho}{\rho_0}\right)^n - 1\right], \tag{21}$$

where ρ_0 - is the density at pressure equal to zero,

$B = 3045\ \mathrm{kg/cm^2}, \quad n = 7.15.$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\frac{\partial v}{\partial r} + \frac{2v\rho}{r} = 0. \tag{22}$$

The velocity of propagation of sound is equal to

$$a = \sqrt{\frac{dp}{d\rho}} = a_0\left(\frac{\rho}{\rho_0}\right)^{\frac{n-1}{2}},$$

where

$$a_0 = \sqrt{\frac{Bn}{\rho_0}} \tag{23}$$

Equations (22) are linear for the case of low pressures. The solutions of these equations which depend solely on $\frac{r}{t}$, when $p_1 = 0$ have the form

$$v = a_0\frac{p}{Bn},$$

$$\frac{r}{t} = a_0\left[1 + \frac{n+1}{2}\frac{p}{Bn}\ln\frac{p_0}{p}\right], \tag{24}$$

where p_0 - is an arbitrary constant.

If a spherical shock wave is propagated in a resting fluid, then at short distances the spread velocity of the shock front will equal

$$N = a_0\left[1+\frac{n+1}{4}\frac{p}{Bn}\right], \quad (25)$$

and the velocity of the particles directly behind the wave front

$$v = a_0\frac{p}{Bn}.$$

Differentiating (24) we have

$$dr = a_0\left[1+\frac{n+1}{2}\frac{p}{Bn}\ln\frac{p_0}{p}\right]dt + a_0 t\frac{n+1}{2Bn}\left[\ln\frac{p_0}{p}-1\right]dp.$$

Together with this at the shock front

$$dr = a_0\left[1+\frac{n+1}{2}\frac{n+1}{4Bn}p\right]dt.$$

To determine the spread velocity of the front, we have consequently the equation

$$\frac{1}{2}p\,dt = p\ln\frac{p_0}{p}dt + t\left[\ln\frac{p_0}{p}-1\right]dp.$$

Whence

$$t = \frac{C}{p\sqrt{\ln\frac{p_0}{p}-0{,}5}},$$

where C - is an arbitrary constant.
Substituting in (24) we obtain

$$r = \frac{Ca_0\left(1+\frac{n+1}{2}\frac{p}{Bn}\ln\frac{p_0}{p}\right)}{p\sqrt{\ln\frac{p_0}{p}-0{,}5}}. \quad (26)$$

We will assume

$$Ca_0 = D\beta\left(\frac{E_0}{\rho_0 a_0^2}\right)^{1/3},$$

where β - is a dimensionless constant.

The expression $\left(\frac{E_0}{\rho_0 a_0^2}\right)^{1/3}$ has the dimension of length.

Then

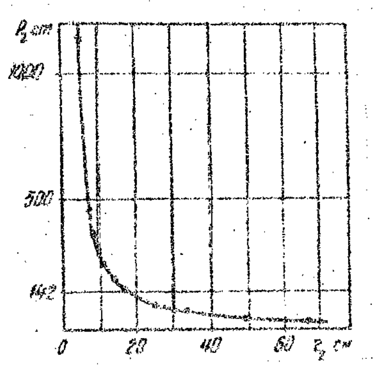
$$\frac{r}{\beta\left(\frac{E_0}{\rho_0 a_0^2}\right)^{1/3}} = \frac{D}{p} \frac{1+\frac{n+1}{2}\frac{p}{Bn}\ln\frac{p_0}{p}}{\sqrt{\ln\frac{p_0}{p}-0,5}} \qquad (27)$$

We will designate $\beta\left(\frac{E_0}{\rho_0 a_0^2}\right)^{1/3} = r_0$.

We assume p_0 = 17000 atm., D = 16200 atm. according to (3).
Then

$$\bar{r} = \frac{r_2}{r_0} = \frac{16200}{p_2} \frac{1+1,87\cdot 10^{-4} p \ln \frac{17000}{p_2}}{\sqrt{\ln\frac{17000}{p_2}-0,5}} \qquad (28)$$

or approximately



$$p_2 = \frac{16200}{\bar{r}} \cdot \frac{1+\frac{4(\ln\bar{r})^{1/2}}{\bar{r}}}{\sqrt{1,15\ln\bar{r}-0,5}} \qquad (29)$$

The parameter of 3 may be determined, knowing p_2 for any single distance r_2 from the center of the detonation with a given E_0. For the above mentioned experiment made at the Moscow Institute of Petroleum and Gas Chemical Industries im. I.M. Gubkin, when $E_0 = 1.87 \cdot 10^{10}$ erg, $\rho_0 =$ = 0.938 g/cm^3, a_0 = 1500 m/sec, β = 0.3, we have r_0 = 0.28 cm.
Thus

$$\bar{r} = \frac{r_2}{0,28} = \frac{16200}{p_2} \cdot \frac{1+1,87\cdot 10^{-4} p_2 \ln\frac{17000}{p_2}}{\sqrt{\ln\frac{17000}{p_2}-0,5}} \qquad (30)$$

The results of computing p_2 according to formula (30) are the following:

r_2, cm 8.4; 11.2; 14.0; 16.8; 25.2; 33.6; 50.4; 67.2; 134.0

p_2, atm. 364.0; 250.0; 188,0; 150.0; 91.0; 65.0; 40.5; 29.2; 19.0.

The calculated dependence of pressure at the shock wave on its radius for this particular experiment is depicted in the figure.

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